An Introduction to Numbers and Their Classifications

All numbers used in mathematics are generally categorized as either real or complex numbers.

Complex Numbers are numbers that can be expressed in the form. $Z = R + iI$, where 'R' and 'I' are real numbers, and " i " is the imaginary unit, defined as $i = \sqrt{-1}$.
Where "R" Is the real part and "iI" It is the imaginary part.
Ex.: $Z = 2 - i3$

1. Imaginary Numbers are the square root or any even root of a negative real number, which are represented by two separate factors: " i " and a real number " I ".
Example of a Complex number: $Z = 3 + \sqrt{-2} = 3 + i\sqrt{2}$
Where: $R = 3$ and $I = \sqrt{2}$ Both are **Real** numbers.
Notes: While expanding polynomials with Complex terms, we should consider that:

$$i = \sqrt{-1} \qquad i^2 = -1$$
$$i^3 = -1i = -i \qquad i^4 = +$$

2. Real numbers "R": include all numbers except imaginary numbers and encompass "Rational" and "Irrational" numbers, as follows.

Rational numbers, denoted by "Q," are defined as numbers that can be expressed as the ratio of two integers, written in the form:

$\frac{m}{n}$, Where m and n are Integers, m, n $\in$ ***Z.*** n $\neq 0$. And $\boldsymbol{Q} \in \boldsymbol{R}$

This ratio could be expressed in decimal form and summarized in one of the following cases:
Ex.: $\frac{-12}{4} = -3$, an integer.
$\frac{19}{8} = 2.375$, an ending decimal.
$\frac{7}{3} = 2.33333\ldots = 2.\overline{3}$, a repeating decimal.
Note: A rational number cannot be an endless decimal, but due to the limitation of the number of digits in calculators, sometimes the end or repeating digits cannot be seen:

Ex.: $\frac{17}{13} = 1.30769230769$ or $\frac{19}{23} = 0.82608695652$

Natural numbers "***N***" are whole numbers used for counting objects:
$N\{0, 1, 2, 3, 4, 5, \ldots\ldots\}$. $\boldsymbol{N} \in \boldsymbol{Z} \in \boldsymbol{Q} \in \boldsymbol{R}$
Note: some sources excluded 0 (zero) from the Natural numbers set, considering:
$N\{1, 2, 3, 4, 5, \ldots\ldots\}$. $\boldsymbol{N} \in \boldsymbol{Z} \in \boldsymbol{Q} \in \boldsymbol{R}$
In such a case, a separate set called **Whole numbers "W"** is defined as:
$W\{0, 1, 2, 3, 4, 5, \ldots\ldots\}$. $\boldsymbol{W} \in \boldsymbol{Z} \in \boldsymbol{Q} \in \boldsymbol{R}$

Integers "*Z*": are positive and negative natural numbers, **Z** = +/ **-** { ***N*** }. ***Z*** ∈ ***Q*** ∈ ***R***

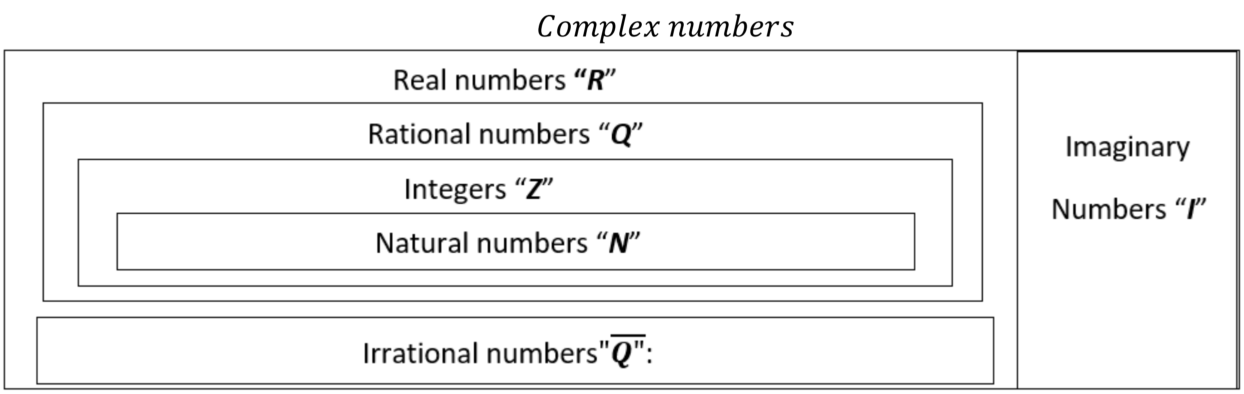
Irrational numbers "$\overline{\boldsymbol{Q}}$": or ***Q'***: are endless decimal numbers, it happens in the case of "numbers which are not a perfect root". $\overline{\boldsymbol{Q}} \in \boldsymbol{R}$

Ex.: $\sqrt{5} = 2.2369679775 \ldots \ldots$, or $\sqrt[3]{4} = 1.58740105197 \ldots \ldots$

This set also includes: π = 3.14159265359… and e =2.71828182846… (natural log base) called **Transcendental Numbers**, this means π and e, are not solutions of any algebraic equation. Unlike the transcendental numbers, non-perfect roots are the solutions of an equation like the following example: the solution for:

$x^2 - 5 = 0$ Is: $x = \pm\sqrt{5}$, With: $x \in \overline{\boldsymbol{Q}} \in \boldsymbol{R}$ Which are not transcendental numbers.

Numbers are summarized in the following diagram:



Prime Numbers:

Prime numbers are natural numbers that are only divisible by 1 and themselves, meaning their only factors are 1 and themselves. 0 and 1 are not prime numbers.

Ex:$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, \ldots.\}$

Factorial:

n! = 1 x 2 x 3 x 4 x 5 x ………. (n - 2) x (n - 1) x n

Ex.: *6! = 1.2.3.4.5.6 = 720*

Note: $1! = 1$ and $0! = 1$

In addition:

$1^n = 1$ for *n* ∈ ***R***; in particular, $1^0 = 1$

$0^n = 0$ if $n > 0$, and undefined if $n < 0$

Important note: $0^1 = 0$ however, $0^0 = 1$ in Combinatorics, although Undefined in Calculus.